QM, non-locality and the vacuum fluctuations: conclusions and new directions

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(partially revised and updated)

We recapitulate on our recent series of papers [1–4] regarding Bell experiments in general, and in particular what we have referred to as "the Wigner-PDC picture of photon entanglement" [11], a formalism that corresponds one-to-one with orthodox QM (QED), at least for PDC (Parametric Down Conversion). Results so far are summarized and placed in context. In addition, once defined the concept of "observable" correlation [28], we make the following proposals:

(i) For a set ψ of "observable" correlations, we insist [4] on the need to substitute the customary description of a state of light $\rho(\psi)$ giving rise to ψ in favor of one that includes explicitly the vacuum degrees of freedom: $\rho(\psi) \rightarrow \rho(\psi, vac)$, this second *opening room* for features that assure no collision with the so-called local-realism (such as variability of the detection probability in the Wigner-PDC model [11]).

(ii) Introducing a distinction between "physical" and "non-physical states", depending on whether they admit or not a proper joint probability distribution for all measurable variables: the "physicality" of $\rho(\psi, vac)$ would guarantee we can observe ψ on a real, physical system.

(iii) In coherence with (i), and at least for light states, in principle there is no reason why we cannot link (once and for all) the "physicality" of a state to its Wigner transform $W[\rho(\psi, vac)]$ being positively defined.

We complete the picture providing directions to pursue in future works:

(iv) A model of atomic/nuclear cascade photon-generation where the interaction with the Zero-Point Field (ZPF) of random fluctuations is explicitly considered, built, in analogy with the PDC case, on a probability density (the Wigner transform) for the field amplitudes of the ZPF modes.

(v) Exploring the relation between the "physicality" of a state and a condition of "statistical equilibrium with the vacuum fluctuations", an equilibrium that seems for us implicit in the foundations of the quantum formalism (not only QED but also bare non-relativistic QM) [38]. Such a relation would be consistent with the fact that, according to (iii), no states with a defined (non-zero) number of photons are "physical": they do not represent an equilibrium state of the fields.

To conclude, we add some comments on the state of the ZPF approach to the physics of massive particles, and why we think the introduction of a frequency cut-off on the ZPF-spectrum may be justified from the physical point of view.

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Previous comments: on "Origin and meaning of quantum nonlocality"

We have recently come across a paper called **"Origin and meaning of quantum nonlocality"** by **De La Peña *et al*** (2011), stating that a model of a free particle immersed in the ZPF background exhibits, upon reduction of the complete phase-space description (where fields in all points of space are considered) to one that models only the particle degrees of freedom, that apparent non-locality.

Quoting from them: **"this reduction of the description from the complete phase space to the configuration space of the particle gives rise to a term representing the fluctuations of the momentum impressed by the zpf on the particle. This term is a function of the probability density $\rho(x)$ and hence possesses a nonlocal nature, manifested even in one-particle quantum systems"** Of course, as far as we understand, once we switch back to the complete description, that non-locality (at least for physically realizable states) disappears. This is exactly our point of arrival, only that in our case we have got there from observation of the Wigner-PDC models: there, even when quantum apparently non-local correlations keep being "observed" (see our definition of "observable" correlation in [28]), local-realism is recovered through the appearance of new observable features such as what we have called *enhancement* and a *reduced detection probability* (see our own account in [4]). Two other comments on their results:

1) From page 13, we quote: *"Of course ours is not the first and only theory from which it follows that the violation of Bell's inequalities does not rule out local realism"*. **Well, the violation of a Bell inequality does** (yes, indeed does, and for any possible theory) **rule out local**

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realism, unless some other additional hypothesis is also violated (that would be the case of a non-genuine inequality - E.Santos). Such is precisely our point of departure here: we need to refine our description of the states of light so room is made for phenomena (for instance, *enhancement* or what we call *reduced detection probability*, see [4]) with the potential to invalidate those supplementary assumptions (not only the potential, we have already shown how it is done [4]). Such room is not present in the ordinary "discrete" Hilbert space description of the states of light, we need to resort to a field-theoretical one. Perhaps they should say, instead: *"Of course ours is not the first and only theory from which it follows that a violation of a genuine Bell inequality (an inequality without supplementary assumptions) is not possible..."* and perhaps add *"as local-realism cannot be violated, because our theory, when transported to full phase-space becomes manifestly local-realistic (before that transportation, it was apparently non local-realistic - only apparently)"*.

2) **Besides, their treatment on the Wigner function may imply some corrections to what we propose here.** They assert that "the Wigner function is not a probability function". Well, it definitely is sometimes, for instance for the vacuum field amplitudes in the Wigner-PDC picture [11] or under certain restrictions: one, we were already conjecturing here, could be statistical equilibrium with the vacuum, which in any case should be enough to achieve a consistent phase space description while reducing the set of relevant degrees of freedom (a "macro-state).

Introductory remarks

In the past few months we have tried to shed some light on a question whose state is, still today, not only confusing, but we will say also harmful, for obvious reasons that we will nevertheless comment later on. Such question was no other than that quantum alleged non-locality. As a result of our efforts, we have posted here a series of papers [1–4] with specific results, as well as another one [6] which did not intended to go further than just a first exploratory attempt. Most of them are still in progress, but even at that stage we think it is quite worthy to recapitulate: there are clear points to remark, and also clear directions where we think someone should try to go forward.

A general conclusion

In spite of a subset of all possible quantum states showing a "non-local" behavior (or what we would choose to call simply a non statistically interpretable one), affirmations such as "QM tells us that nature is non-local" do not hold up against, not only common sense, but neither against a serious scrutiny of decades of experimental evidence. Such evidence confirms the correctness of quantum predictions, yes, but also points in yet another direction: the need to refine our framework. The Wigner-PDC picture shows that such refinement may simply consist in the adoption of a field-theoretical description of light, within the formalism of QED.

Our criticism on "non-locality": semantic and content

Our position on the non-locality issue goes along two main lines: one criticizes the formal approach to the question, which we find misleading; the second is based on our own, pretty exhaustive analysis of the experimental evidence. Such analysis was confined, nevertheless, to theoretical grounds, and we are the first to admit that it may have the expectable limitations, though we are at this point well convinced that, whichever they may be, they would not change our conclusions here. Besides, what we mean by "exhaustive" is that we have not left any gap or missing piece along the way (which would be irresponsible from us): at least as far as our knowledge goes, everything fits.

(A) In the first place, with theorems forbidding the use of shared entanglement to support superluminal transmission of information [26], the term "non-local" is rather arbitrary: from our point of view it should be, at least in most occasions, substituted by something like *"incompatibility with the axioms of probability"*.

Indeed, roughly speaking, Bell inequalities are nothing but reformulations of the axiomatic laws of probability (the ones formulated in terms of correlations can be mapped to the probability formulation, some comments on this being due anyway [29]): the predictions of QM for the results of certain measurements on certain states do not seem to abide by such laws [27], unless we allow for some sort of action-at-a-distance, enabling the measuring devices to accommodate their results in a suitable way. It is then true that some of those inequalities, like for instance the original one by J. Bell [30], include supplementary assumptions in addition to bare probability axioms, such as that intuitive form of "locality", introduced through independence of random variables defined in space-like separated locations; yet, QM keeps on defying other Bell inequalities when such space-like separation is not assumed as a physical feature of the experimental setup. For instance, nor the Clauser-Horne-Shimony-Holt [31], neither the Clauser-Horne [32] inequality assume such independence, explicitly or implicitly, anywhere: we can obtain them from the axioms of probability alone.

Our message, then, is that collision with the classic probability framework is the primary issue: to associate

it with some mysterious sort of “non-locality” involves adding new hypothesis to the picture. For lack of a better word, we will nevertheless use the term “collision with local-realism” as a short-hand for what we have described in the last paragraphs: from our point of view somehow an intermediate one, not completely accurate either, but at least neutral in what regards the interpretation of the question.

(B) The fact that some entities (some “states”) allowed within the framework of QM may show such inconsistency with the axioms of probability (what others would call a sort of “non-locality”)... does not mean, unless we really do a demanding exercise of biased imagination, that actual physical states with such property must exist at all. So far, more than four decades of (very necessary and very useful) experimental work has proven that, if such states do exist, they are really hard to find. With more detail, what they confirm is the following: let us label by ψ a set of correlations (marginal results, bipartite or n-partite correlations), and $\rho(\psi)$ a state for which QM predicts ψ:

1) when we test the possibility of preparing a physical system in $\rho(\psi)$, in order to observe ψ (it is then convenient to define the concept of “observable correlation”, what we have done formally in [28], as a correlation “conditioned to the subensemble where measurements yield all the necessary proper results”),

2) what we find is: we can indeed “observe” correlations as specified by ψ, but the physical state on which this is possible also shows other additional features, such as “enhancement” (see [1] and side-notes of [2]) and what we would call, in place of “detector inefficiency”, as “reduced detection probability” [4].

Such features invalidate the supplementary assumptions which are always present (at least one of them) in every and each experimental test of a Bell inequality carried out so far, rendering the violation of the inequality to something meaningless. Those supplementary assumptions are:

(i) **fair-sampling** or any equivalent one [34], for the case of *homogeneous* inequalities (no marginal results or probabilities involved, see [8]),

(ii) **no-enhancement** [35], for the case of *inhomogeneous* inequalities in their *non-genuine* formulation (non-genuine because it involves the no-enhancement hypothesis),

(iii) **detection probability beyond the critical threshold**, given by the so-called “critical efficiency” [36], this last applying both to *homogeneous and inhomogeneous* inequalities, either in their *genuine or non-genuine* version.

For the sake of completeness, we must also comment on the so-called ”All versus Nothing” (AVN) proofs of the incompatibility of QM with local-realist theories: their arguments involve, simultaneously, a set of observables that cannot be simultaneously measured on the same system. Bell inequalities operate on ensembles which in principle eliminates that difficulty: average values or probabilities can be, again in principle (under certain hypothesis that we have already seen are not satisfied on real experiments), estimated simultaneously on the same ensemble; indeed, a Bell inequality can be obtained from each AVN proof. Anyway, if somehow we could check that the suitable theoretical state (upon which the corresponding AVN proof is implemented) is properly prepared with massive particles (we have already seen that this is not the case of photons), AVN proofs would still retain value as proofs that spin-projections cannot correspond to “elements of reality”.

A brief summary of our work

In regard to the former three assumptions: in the first place, we were well acquainted with (i) due to our previous work in the problem of critical efficiencies for ”homogeneous” Bell inequalities [37]; secondly, we investigated (ii) and (iii), typical of “inhomogeneous” inequalities, in [1], as well as the physical origin of - a breaking of - (ii) in [2] (as a side-note).

In addition to that, also in [2] we presented what we believe an original proposal on how to overcome the “detection theory problem” in the Wigner-PDC framework [11], that allegedly rendered such approach to a dead-end alley. This proposal is important because an entirely local model of PDC-generated photon entanglement adds empirical support to all the former proposals, so far formulated as a merely abstract framework of ideas: the Wigner-PDC model predicts, amongst other phenomena, those of *enhancement* and a *reduced detection probability*, necessary to invalidate (ii) and (iii), respectively, and of course it also gives rise to predictions that “break” the fair-sampling assumption in (i), which is no surprise because all the supplementary hypothesis are intimately related, see [1]. Final conclusions about (i), (ii) and (iii) were presented in [4]: at the light of Wigner-PDC optics, the usual “critical efficiency game” is relegated to something close to absolute irrelevancy, at least as far as PDC-generated photon entanglement is concerned.

To complete the picture we will add that, previously, we had also explored some alternative routes: for instance, in [5] we proposed a local model for the singlet correlations that later we realized seemed clearly ruled out by Weihs ’98 experiment. All this said and even at this stage, we do not regard our work as a point of arrival, but rather as a departure one from where somebody could acquire a perspective, a well-founded one, that may be useful for further work on all these issues.

The need for a change of framework

In this situation, the **Occam's razor** logic is not "detectors and every other device in the experimental setup conspire against us in every conceivable way, so we cannot obtain a conclusive proof of what we want"; the right one is "let us try to refine our description of the physical state of real systems so that, aside from accounting for the observed set of correlations, we account as well for all those additional features that are also experimentally observed (and that prevent any inconsistency with local realism)". Even in the (our) worst case scenario, predictions based in a more complete description might cast light on what is really going on.

The search for that enhanced description becomes even more justified when we realize that it can perhaps be simply obtained from an extension of the foundational quantum formalism: it suffices to adopt Quantum Electrodynamics (with second quantization of the fields) as our new framework, such is at least what the Wigner-PDC picture [11] seems to suggest. We advocate a substitution

$$|\psi\rangle \in \mathcal{H}_n \rightarrow |\psi, vac\rangle \in \mathcal{H}_{n,vac}, \tag{1}$$

or, in the most general case, with density matrices, $\rho(\psi) \rightarrow \rho(\psi, vac)$, where we recall we are using ψ merely as a "label" that denotes a set of observable correlations, and where:

(i) $\mathcal{H}_n$ denotes the Hilbert space where the state of (the spin of n) photons is represented: we were therefore treating photons just as we would do with half-integer spin massive particles such as electrons, as it is customarily done within the field of Quantum Information (therefore it is a finite-dimensional representation of such state, with $\dim \mathcal{H}_n = 2^n$ for the non-relativistic case).

(ii) we are redefining our overall "space of representation" from $\mathcal{H}_n$ to $\mathcal{H}_{n,vac}$ (we are deliberately choosing to use this kind of abstract terminology, see [6]) as

$$\mathcal{H}_{n,vac} \equiv \mathcal{H}_n^{(ext)}, \tag{2}$$

where $\mathcal{H}_n^{(ext)}$ denotes the description of the n particles as fields, therefore a description "extended" to cover the whole coordinate space, that already includes (through the commutation relations, either for field operators, either for the equivalent creation/annihilation ones) the vacuum degrees of freedom: random amplitudes of the fields at each point of space, something absent in the previous picture (we will comment more on this anyway). $\mathcal{H}_n^{(ext)}$ is therefore infinite-dimensional, and nothing but the arena where QED operates, as well as the base upon which the Wigner-PDC picture of photon-entanglement [11] is built.

In principle (only in principle), $\mathcal{H}_n$ could provide a valid description if we could somehow decouple the state of light we want to describe from the random fluctuations of the fields. The field theoretical description of the fields introduces a random component (in other words a probability of transition) that makes that decoupling impossible, unless we assume some additional hypothesis, for instance compatibility of our state with the *statistical equilibrium of the random background* [38]: in that case we are necessarily assuming an statistical description (in other words we work with a "macro-state"), which is on the other hand what the quantum formalism seems to be build for. In those conditions, we can describe our state as a system with discrete degrees of freedom, though what we really have, behind that description, is a set of fields for each point of space; this, whether the state is more or less localized: we can regard the n photons as particles with localized positions, as non-localized plane waves with neatly defined momentum, or any other intermediate case.

From the point of view of Wigner-PDC optics, such a decoupling is precisely what experimental evidence is telling us that *does not happen*; again, such thing should hardly be a surprise given that, (i) a state of n photons (localized or not) clearly does not represent an state of equilibrium of the vacuum fluctuations, and, of course, (ii) we already know that many of those states do not yield (once physical space-like separation constraints are introduced in the picture, or even without them, in the case of the Wigner transform not positive definite) predictions consistent with the axioms of probability.

Summarizing, the Wigner picture of PDC-Optics [11] shows that suitable states $\rho(\psi, vac)$ give rise to all the (observable) correlations in the set ψ, and at the same allow room to accommodate the additional features that are necessary to prevent any inconsistency with local realism (or let us say, in coherence with the approach we are advocating here, the axiomatic probabilistic framework). The set of experiments explained from this approach includes not only polarization-entanglement schemes but also others schemes; within the first series of papers [12–18] those experiments included: frustrated two photon creation via interference [12]; induced coherence and indistinguishableness in two-photon interference [12, 13]; Rarity and Tapster's 1990 experiment with phase-momentum entanglement [13]; Franson's (original, 1989) experiment [13]; quantum *dispersion cancellation* and Kwiat, Steinberg and Chiao's *quantum eraser* [14]. From the most recent series [19, 20] we can add: two-qubit entanglement and cryptography [19]; quantum key distribution and eavesdropping [19]. We have also had access to some promising (but still unpublished) work on *hyper-entanglement* [21].

A proposal: *physical* and *non-physical* states

Though we are well aware that this picture does not (yet) generalize to all schemes of photon generation, such as atomic/nuclear cascades, we still think it is justified at this point (and may be useful) to introduce the following

definitions. As we have done sometimes in the past, we will not be too mathematically formal, for the sake of brevity and because it is, for now, not really necessary; we are at a barely exploratory stage.

Proposal 1 - Def.: *A quantum state ρ is a* physical state *whenever a well-defined joint probability density function can be build for the results of the possible measurements that could be performed on the system it describes,* non-physical *on the contrary. The set of possible outcomes of those measurements may include the "non-detection" as just another proper result (see our comments in [40]).*

As given, that first definition depends both on the state itself... and what set of (measurable) variables we choose to consider (relevant for the description of the system). In the case of light states, we can eliminate this ambiguity by resorting to the so-called Wigner quasi-probability distribution [9, 10] ("quasi" stands for the fact that it is not positively defined for every state: a fact upon which we will base our following proposal). We have to be nevertheless aware that the Wigner function only characterizes position and momentum, or any other pair of conjugated variables obtained as combinations of those two (for instance, complex amplitudes of fields and their conjugates, corresponding to the creation/annihilation operators in the second-quantized description). This leaves aside variables corresponding to other degrees of freedom such as projections of spin; nevertheless, such thing is irrelevant of light states [39]: light can be completely described by giving the values of the fields at each point of space.

Proposal 2 - Def.: *A state ρ is physical iff its Wigner transform is positively defined [10].*

One of the reasons why this last proposal is interesting is the fact that, for all light states with a fixed, non-zero number of photons (similar to energy eigenstates of a harmonic oscillator), the Wigner transform is not positively defined: this is again telling us that our "discrete" Hilbert space is not suitable for representing physical states of light!

On why the present state of the question is not only confusing, but also harmful

We began this article by saying that the present state of the non-locality question is not only confusing but harmful. The reason why we say this is quite evident: quantum alleged non-locality is always invoked (consciously or subconsciously) as the primary barrier why alternative efforts, such as those recently presented in [22] (some of whose ideas we were independently suggesting in [6], although at a much more preliminary stage of development), or all these other concerning the Wigner-PDC approach, are routinely deemed superfluous. The truth, for whoever wants to dedicate it enough attention, an unbiased one, is quite different: there is no experimental evidence of any sort of non-locality in nature. Moreover, the data we have, though agreeing perfectly with quantum predictions (which is rewarding of course), is clearly also pointing in a different direction, the need to refine the framework where we study the states of physical systems, not only for light alone (photons), where most efforts have been done, but also for massive particles. And, as one can see from the Wigner-PDC approach, there is plenty of room to do such refinement within the quantum formalism itself.

On future directions

(a) In the first place we will point out the need for explaining, on similar grounds (the use of Wigner's function), other schemes of photon-entanglement generation, in particular those that use atomic/nuclear de-excitation cascades.

(b) Another natural line of research would consist on exploring the already mentioned condition of "statistical equilibrium with the vacuum fluctuations" as a necessary (or non-necessary) hypothesis for the "physicality" of a given state [38].

Last comments

The ZPF background plays, in some ways, a similar role to that of thermal noise in mechanical systems: it can be ignored unless we want to observe dynamics at a microscopic scale (so is the case, for instance, of phenomena such as *spontaneous emission* in the presence of a sufficiently intense external potential). There is a difference, anyway: in the case of purely mechanical systems, macrodynamics is oblivious to thermal noise, while in electromagnetic ones, whether we choose or not to include the ZPF into the model, certain features of the macroscopical dynamics, such as typically quantum ensemble behavior, are already in great part implicitly determined by the existence of this background, a necessary condition for some of the basic assumptions (quantization of angular momentum) that shape the structure of the mathematical formalism of the theory itself: such is at least our conjecture, see [6].

We will also add some comments on the state of the ZPF approach to (I) the physics of massive particles, and (II) why we think the introduction of a frequency cutoff on the ZPF-spectrum (not strictly necessary for the mathematical coherence of the theory, but convenient) may be justified from the physical point of view.

About (I), the massive-particle case is not at all unexplored territory, with some recent (and promising) advances. For instance, the physics of ensembles of massive particles as driven by the interaction with the ZPF background of vacuum fluctuations has been recently treated, extensively, in [22], in particular some of the lines that

we had suggested, at a much earlier stage and ignoring the existence of that paper, in our exploratory effort of [6]. Besides, a mechanism of entanglement between massive particles as a coupling through ZPF-modes has also been proposed in [23]. Such mechanism provides an important piece to the puzzle, because so far, only photon-entanglement had been explained in a similar fashion, through what we have been calling the Wigner-PDC picture.

In respect (II), to our knowledge the absence of a frequency cut-off on the ZPF-spectrum does not compromise the mathematical coherence of the sort of ZPF-based stochastic mechanics that we are advocating here: indeed, under very basic hypothesis (temporal and spatial integration, plus a "low-band" frequency response from physical systems), the net of the ZPF background to the observable electromagnetic spectrum can be regarded as null or negligible (see side-note in [2], for instance). However, from the physical point of view, the appearance of such cut-off is clearly more than desirable, to avoid for instance the embarrassment of infinite energy; our vision on this is summarized in the following points:

(i) In the first place, the ZPF ω^3-spectrum is forced by QED, which means a combination of QM with special relativity,

(ii) Our building brick for QM is "quantization of angular momentum" [6], expressing the fact that for a charged particle, any periodic or quasi-periodic trajectory must satisfy a balance of radiated/absorbed power that restricts the value of the (magnitude and projections) of such angular momentum to a discrete spectrum.

(iii) Trajectories like those of (ii) require, as a necessary hypothesis, an state of equilibrium with the vacuum fluctuations, but also, from the point of view of classical system dynamics, the intervention of "inner degrees of freedom", so as to provide, through a higher frequency feedback dynamics, an stabilizing mechanism for the dynamics at the lower frequency-range. This issue has been explored by us, elsewhere, in relation with the stability of atomic orbitals [7].

(iv) Point (iii) implies that the structure of physical systems (so far descending from the molecular/atomic realm to the nuclear and even sub-nuclear one) imposes a limit on which frequencies can be attained by those metastable periodic trajectories, so therefore it is natural to think that, out of a certain range, the main assumption implicit in QM (according to us, stability and therefore quantization of angular momentum) may not be justified anymore.

(v) Our points (iii)-(iv) seem to be consistent with Puthoff's proposed solution for the problem of the origin of the ZPF [24], where the vacuum fluctuations are the result of the micro-oscillations of matter (that we can assume here as a postulate or as a prediction of QM); according to us [7], the existence of a high-frequency range of those micro-oscillations is the element that allows for the existence of metastable states, and their low-frequency transitional dynamics, which is the one that QM models and the only one we can observe, at least with our present measurement-technology.

In any case, whether it is or not justified by the former points, and may them be correct or not (they are nothing more than conjectures at this stage), setting a frequency cut-off is the customary procedure (and still the only one to our knowledge) to get rid of infinities in QED: all we say is it should lead us to seriously wonder why.

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[3] D. Rodríguez, "Revisiting factorability and indeterminism". ArXiv.

[4] D. Rodríguez, "Bell tests with photon-entanglement: LHV models and critical efficiencies at the light of Wigner-PDC optics". ArXiv.

[5] D. Rodríguez, "Communication loophole in a Bell-EPR-Bohm experiment: standard no-signaling may not always be enough to exclude local realism". ArXiv.

[6] D. Rodríguez, "A classical, elementary approach to the foundations of Quantum Mechanics". ArXiv.

[7] D. Rodríguez, "Orbital stability and the quantum atomic spectrum from Stochastic Electrodynamics". Arxiv.

[8] *We now realize that the distinction between homogeneous and inhomogeneous inequalities (ref) is more convenient than what it used to be our choice: inequalities written in terms of correlations, all of them homogeneous, and inequalities written in terms of probabilities.*

[9] Y.S. Kim, M.E. Noz. "Phase space picture of Quantum Mechanics" (Group theoretical approach), Lecture Notes in Physics Series - Vol **40**, ed. World Scientific.

[10] The **Wigner transform** (see for instance [9]) applies a state ρ in a real, multivariable function:

$$W[\rho] : \rho \rightarrow W(\{x, p\}_{\mathcal{H}}) \in \mathcal{R}, \tag{3}$$

where the set $\{x, p\}_{\mathcal{H}}$ of variables upon which $W(\cdot)$ takes values is obviously depending on to the structure of $\mathcal{H}$, the so-called Hilbert space of the system.
For instance, for the most simple description of a system implemented in QM, a particle living in an one-dimensional space, with wavefunction $\Psi(x, p)$, we would have

$$W(x,p) = \frac{1}{\pi\hbar} \int \Psi^*(x+x') \cdot \Psi(x-x')\; e^{\frac{2ipx'}{\hbar}}\; dx', \tag{4}$$

where p is shown to correspond to the actual "linear momentum", and where $\Psi^*(x+x') \cdot \Psi(x-x')$ can be interpreted as a matrix element from a density matrix ρ_x

defined on the position basis:

$$[\rho_x]^{x+x'}_{x-x'} = \Psi^*(x+x') \cdot \Psi(x-x'), \tag{5}$$

which already gives us clues about how to generalize it to a density matrix built on other degrees of freedom (for instance spin):

$$W(x,p) = \frac{1}{\pi\hbar} \int \langle x+x'|\rho|x-x'\rangle \; e^{\frac{2ipx'}{\hbar}} \; dx'. \tag{6}$$

Whenever $W(x,p) \geq 0 \; \forall x, p$, $W(x,p)$ is a joint probability density for x, p, but still, if this is not true, well-defined marginal probability densities for either x and p can be obtained from it, integrating in the corresponding degree of freedom.

So far, in the most general case, other degrees of freedom of the system, such as spin projections, lay out of the picture; however, this is not the case of light, where field amplitudes can be expressed as combinations of x and p, modeling the state of fields as the state of a set of harmonic oscillator, one for each point of space.

This last is the reason why we defend the convenience of, in the case of light, associating the positivity (or non-positivity) of the W-function to the physical (or non-physical) character of the state.

Therefore, with "positively defined" we mean, in the most general case, $W(\{x,p\}_{\mathcal{H}}) \geq 0$, for all possible values of the set of parameters $\{x,p\}_{\mathcal{H}}$ of the transformation.

[11] The **Wigner-PDC picture** describes photon entanglement generated from Parametric Down Conversion (PDC). The list of typically quantum experiments explained by this approach, and even more the simplicity which with it is done, is impressive. The formalism is developed in the following series of papers [12–18], and recently revitalized with another stream of interesting results [19, 20]. A consistent theory of detection is not completed in any of this papers: that is what we have tried to in [2].

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[21] A. Casado, private communication.

[22] L. Fritsche, M. Haugk. "Stochastic Foundation of Quantum Mechanics and the Origin of Particle Spin". ArXiv, perhaps published already too.

[23] A. Valdés-Hernández, L. de la Peña and A. M. Cetto. "Bipartite Entanglement Induced by a Common Background (Zero-Point) Radiation Field", Foundations of Physics **41**, 843-862 (2011).

[24] H.E.Puthoff, "Source of vacuum electromagnetic zero-point energy", Phys. Rev. A, **40** 4858-4862 (1989).

[25] Whether the Wigner-PDC picture of photon entanglement can be interpreted or not as a "local-realistic" theory (or, let us say, in coherence with our approach here, whether it is consistent with all probability axioms, something that we have tried to solve in [2]), a partial consequence to obtain from it is that *the photon is not, at least necessarily, a particle.*

[26] **Theorems that forbid superluminal transmission using QM: "no communication theorems".**

[27] **On Bell inequalities and the classic probabilistic framwork:**
If Λ is the a space of events, all possible probability functions within the formalism, $F(\cdot) : \lambda \in \Lambda \to F(\lambda)$, satisfy $0 \leq F(\lambda) \leq 1 \; \forall \lambda$ and, for any partition $\{\Lambda_i\}$ of Λ (assuming of course additivity on disjoint subsets of events), $\sum_i F(\Lambda_i) = 1$.
All Bell inequalities (either the ones written in terms of probabilities or the ones that are written in terms of correlations) can be obtained from as a derivation from those laws, plus some very simple supplementary assumptions: statistical independence of variables defined at distant locations. Many do not need any supplementary ("locality") assumptions: such is the case of both the Clauser-Horne inequality (1974) within the probability case, or the Clauser-Horne-Shimony-Holt one (1969) within the "correlations" one.
A chosen set of probabilities can be represented in a multidimensional coordinate space, so that now the set of axiomatic restrictions and all relevant recombinations of them (amongst them some that we may know as "Bell inequalities") define a closed region within that space: a "*no-signaling polytope*".

[28] **Defining observable correlations:**
It is crucial in our discussion to introduce the concept of *observable correlation*: while the most general set of correlations over a set of measurements $\mathcal{M}$ (for simplicity let us take all them dichotomic, the results different from zero) can be simply specified by a set of average values $\langle M_i \cdot M_j\rangle$, for all pairs $M_i, M_j \in \mathcal{M}$, we will only call *observable* the one specified by a set of averages $\langle M_i \cdot M_j | M_i, M_j \neq 0\rangle$, where $M_i \neq 0$ indicates that a detection (a proper result) is obtained when the observable M_i is measured.
For instance, if we consider that '0' is a valid outcome (even when it corresponds to an absence of detection), or if we choose to assign any other value to that case (as it is indeed done in some experimental setpus, quite recent ones - add refs.), then the concepts of bare *set of correlations* and *set of observable correlations* clearly differ. Besides, our former restriction to bipartite correlations is done for convenience (the most general set can contain from marginal values $\langle M_i | M_i \neq 0\rangle$ to n-partite correlations: $\langle M_1 \cdot \ldots \cdot M_n | M_1, \ldots M_n \neq 0\rangle$.

[29] **On Bell inequalities written as probabilities or as correlations:**
Mathematically, the map may be one-to-one, but *experimentally* (at an actual experimental test), the choice of one formulation or another involves different procedures, which in turn involve different supplementary assumptions. In particular, if we evaluate a correlation-type in-

equality, we need to assume *fair sampling*, while with a probability based inequality, we usually assume *no-enhancement* (this is the non-genuine version of the inequality) or simply that the detection probability surpasses the critical threshold ($\eta_{\rm crit}$), for what we would call the *genuine* version of the test.
This is so even when the data-set might be the same (it is difficult to believe we got to be told something like "come on, current experiments measure everything at the same time"), **because *such supplementary assumptions involve how the data-set is to processed in each case***: which data must be rejected (fair-sampling) or how the probabilities are estimated (because they are estimated) from those data (no-enhancement).

[30] **Original Bell inequality:** J.S. Bell, "On the Einstein-Podolsky-Rosen paradox", Physics (Long Island City, N.Y.) **1**, 195 (1964).

[31] **CHSH inequality:** J.F. Clauser, M.A. Horne, A. Shimony, and R.A. Holt, "Proposed experiment to test local hidden-variable theories", Phys. Rev. Lett. **23**, 880 (1969). Let A_1, A_2 and B_1, B_2 be, respectively, pair of dichotomic ($A_i, B_j \in \{\pm 1\}$) observables at two distant sides, we write

$$|\langle A_1 B_1\rangle + \langle A_1 B_2\rangle + \langle A_2 B_1\rangle - \langle A_2 B_2\rangle| \leq 2.$$

As opposed to the original Bell inequality in [30], this inequality - it can be proven - remains valid for indeterministic theories (loss of information in the measurement process).

[32] **CH inequality:** J.F. Clauser, M.A. Horne, "Experimental consequences of objective local theories", Phys. Rev. D **10**, 526 (1974). Let A_1, A_2 and B_1, B_2 be, respectively, pair of dichotomic ($A_i, B_j \in \{\pm 1\}$) observables at two distant sides, a possible formulation (there are equivalent ones) is

$$\begin{aligned} &P(A_1 = B_1 = 1) + P(A_1 = B_2 = 1) + P(A_2 = B_1 = 1) \\ &\quad -P(A_2 = B_2 = 1) - P(A_1 = 1) - P(A_2 = 1) \leq 0, \end{aligned} \tag{7}$$

This inequality considered, explicitly, indeterminism (although [31] includes this case as well). Besides, we provide the operational expression used in experiments on this inequality (see, for instance, [33], eq. 32-33). Estimating, from the "number of counts" registered:

$$P(A_i = 1, B_j = 1) \approx N(a_i, b_j)/N(\infty, \infty), \tag{8}$$

$$P(A_i = 1) \approx N(a_i, \infty)/N(\infty, \infty), \tag{9}$$

$$P(B_j = 1) \approx N(b_j, \infty)/N(\infty, \infty) \tag{10}$$

and substituting in [32] we obtain the inequality $S_{(exp)}(CH)$, with

$$\begin{aligned} &S_{(exp)}(CH) \\ &= (N(a_1, b_1) - N(a_1, b_2) + N(a_2, b_1) + N(a_2, b_2) \\ &\qquad -N(a_2, \infty) - N(\infty, b_1))/N(\infty, \infty), \end{aligned} \tag{11}$$

where, in relation to [32], $a_i \equiv A_i = +1$, $b_j \equiv B_j = +1$ and ∞ means the photon is detected with no polarizer or device placed on its way. On the other hand, to estimate there the marginal probabilities we are assuming *no enhancement*: probability of detection cannot be enhanced by placement of a polarizer between the source and the detector. See also [35].

[33] A very nice account of photon Bell experiments by Alain Aspect himself:
A. Aspect, "Bell's theorem: the naive view of an experimentalist", in "Quantum (Un)speakables - From Bell to Quantum Information", ed. by R.A. Bertlmann and A. Zeilinger, Springer (2002).

[34] **Fair-sampling.** From Clauser, Horne, Shimony and Holt's well known '69 paper: "given a pair of photons emerges from the polarizers, the probability of their joint detection is independent of the polarizer orientations".

[35] **No-enhancement.** From Clauser and Horne's well known '74 paper: "for every emission λ, the probability of a count with a polarizer in place is less or equal to the probability with the polarizer removed".

[36] **Critical efficiencies:** For a detection probability superior to $\eta_{\rm crit}$, no local hidden variables can exist... for the contrary case such model may exist and the (experimental) violation of the inequality is meaningless.

[37] **Previous work on LHV's and critical efficiencies:**
(i) A. Cabello, D. Rodríguez, I. Villanueva. Phys. Rev. Lett. **101**, 120402 (2008),
(ii) A. Cabello, J. -Å. Larsson, D. Rodríguez. Phys. Rev. A **79**, 062109 (2009).

[38] **Bare *statistical equilibrium*:** "all micro-states compatible with a given macro-state are equiprobable".
Thermal equilibrium is defined (and a parameter of temperature introduced), obviously, beyond this primary equilibrium with the ZPF (T=0). Implicit in that definition is the fact that any equilibrium is defined *on a set of degrees of freedom* (so it may leave others aside, in this case thermal equilibrium leaves aside the vacuum fluctuations degrees of freedom).
Clearly, within the QED framework, states such as those of n-photons do not represent an equilibrium state of the random background. If we accept this equilibrium as one of the foundational hypothesis of QM (and we have reasons for that, as such equilibrium is necessary for the angular momentum "quantization" that we have advocated as the building brick of the theory in [6]), then it is not so surprising that such states may yield an *unphysical* (see the main text) set of predictions.

[39] **Light and spin:** Light (or the photon, as the quantum orthodoxy would say) does not have "spin", but it carries angular momentum (or polarization-states): due to the particularity of its propagation velocity, such momentum is never transversal to the propagation direction, and the set of possible states accepts an isomorphism with the $SU(2)$ group of half-integer spin massive particles - such as a non-relativistic electron. We may use the shorthand "photon-spin" for convenience anyway.

[40] As a matter of fact, in *physical* states one can always find a well-defined probability distribution for the values of projections of "photon-spin" (as "hidden variables")... whenever we admit for the "absence of detection" (the '0' hidden instruction, in common terminology when dealing with local hidden variable models) as just another proper value. This is precisely what Bell experiments are proving, over and over.